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Duplicating a Vertex with an Edge in Divided Square Difference Cordial Graphs

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ABSTRACT

In this present work, we discuss divided square difference (DSD) cordial labeling in the context of duplicating a vertex with an edge in DSD cordial graphs such as path graph, cycle graph, star graph, wheel graph, helm graph, crown graph, comb graph and snake graph.

1. Introduction

By a graph, we mean a finite, undirected graph without loops and multiple edges. For basic definitions we refer (Harary 1969). In 1967, (Rosa, 1967) introduced a labeling of G called β-valuation. A dynamic survey on different graph labeling was found in Gallian (Gallian, 2008). Cordial labeling was introduced by (Cahit, 1987). (R. Varatharajan, 2011) have introduced the notion of divisor cordial labeling. (Alfred Leo, 2018) introduced divided square difference cordial labeling graphs. (Kaneria, 2016) introduced balanced cordial labeling. The motivation behind the divided square difference cordial labeling is due to R. Dhavaseelan et.al on their work even sum cordial labeling graphs (R. Dhavaseelan et. al, 2015). The motivation behind this article is due to S.K. Vaidya et.al on their work (S. K. Vaidya et. al, 2012). In this present work, we discuss divided square difference (DSD) cordial labeling in the context of duplication of a vertex by an edge in DSD cordial graphs such as path graph, cycle graph, star graph, wheel graph, helm graph, bistar graph, crown graph, comb graph and snake graph.

2. Preliminaries

Definition 2.1 (Galian,2008)

Graph labeling is an assignment of numbers to the edges or vertices or both subject to certain condition(s).

Definition 2.2 (Cahit,1987)

A binary vertex labeling f of a graph G is called a *Cordial labeling* if $|v_f(0)-v_f(1)|\le 1$ and $|e_f(0)-e_f(1)|\le 1$.

A graph G is cordial if it admits cordial labeling.

Definition 2.3 (Kaneria et.al., 2016)

A cordial graph G with a cordial labeling f is called a balanced cordial graph if

$$|e_f(0)-e_f(1)|=|v_f(0)-v_f(1)|=0.$$

It is said to be *edge balanced cordial graph* if

$$|e_f(0)-e_f(1)|=0 \text{ and } |v_f(0)-v_f(1)|=1.$$

Similarly it is said to be *vertex balanced cordial graph* if

$$|e_f(0)-e_f(1)|=1 \text{ and } |v_f(0)-v_f(1)|=0.$$

A cordial graph G is said to be *unbalanced cordial graph* if

$$|e_f(0)-e_f(1)|=|v_f(0)-v_f(1)|=1.$$

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Definition 2.4 (Alfred Leo, 2018)

Let $G=(V,E)$ be a simple graph and $f:V\rightarrow\{1,2,3,\ldots|V|\}$ be a bijection. For each edge, assign the label 1 if $\left|\frac{(f(u))^2-(f(v))^2}{f(u)-f(v)}\right|$ is odd and the label 0 otherwise. f is called divided square difference cordial labeling if $|e_f(0)-e_f(1)|\leq 1$, where $e_f(1)$ is number of edges labeled with 1 and $e_f(0)$ is number of edges labeled with 0.

A graph G is called DSD cordial if it admits DSD cordial labeling.

Definition 2.5 (Alfred et.al, 2018)

A divided square difference cordial graph G is called a *balanced DSD graph* if $|e_f(0)-e_f(1)|=0$.

A divided square difference cordial graph G is called a *unbalanced DSD cordial graph* if $|e_f(0)-e_f(1)|=1$.

Proposition 2.6 (Alfred Leo, 2018)

1. Any path P_n is a DSD cordial graph.
2. Any cycle C_n is a DSD cordial graph except $n\equiv 2(mod 4)$.
3. The star graph $K_{1,n}$ is a DSD cordial.

Proposition 2.7 (Alfred Leo et.al., 2018)

1. The wheel graph $W_n,(n\equiv 0,1(mod 4))$ is DSD cordial
2. The helm graph $H_n,(n\equiv 0,1(mod 4))$ is DSD cordial.

Proposition 2.8 (Alfred Leo et. al., 2018)

1. The crown graph $C_n\odot K_1$ is DSD cordial.
2. The comb graph $P_n\odot K_1$ is DSD cordial.

Proposition 2.9 (Alfred et. al., 2018)

The triangular snake graph T_n (except $n\equiv 3\text{mod}4$) is a balanced DSD cordial when n is odd.

3. Main Results

Proposition 3.1

A graph got by duplicating a vertex v_k with an edge $e'(=u'v')$ in a DSD cordial path P_n (except $n\equiv 2(mod 4)$) is DSD cordial.

Proof

Let G be a path graph P_n (except $n\equiv 2(mod 4)$). By Proposition 2.6, we draw a DSD cordial path P_n. Now, we duplicate any of the vertex v_k in G with an edge $e'=u'v'$ and construct a new graph G'. In this graph G', $|V(G')|=n+2$ and $|E(G')|=n+2$. For DSD cordial labeling pattern, let the vertex labels are $\{1,2,\ldots,n+2\}$. Then, by labeling u' and v' by $f(u')=n+1$ and $f(v')=n+2$, we get $|e_f(0)-e_f(1)|\leq 1$.

Hence G' is also a DSD cordial.

Remark 3.2

From Proposition 3.1, in particular we get $|e_f(0)-e_f(1)|=0$ when n is even.

Hence, we can conclude that G' is a balanced DSD cordial graph when n is even and unbalanced DSD cordial when n is odd.

Note 3.3

For $n\equiv 2(mod 4)$, the path graph P_n is DSD cordial whereas G' obtained by duplicating any of the vertex with an edge in P_n is not DSD cordial.

Example 3.4

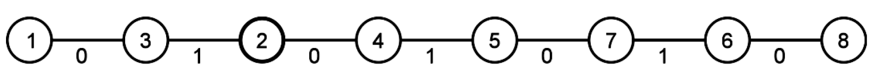

Fig 1(a) Unbalanced DSD cordial graph $G(P_8)$

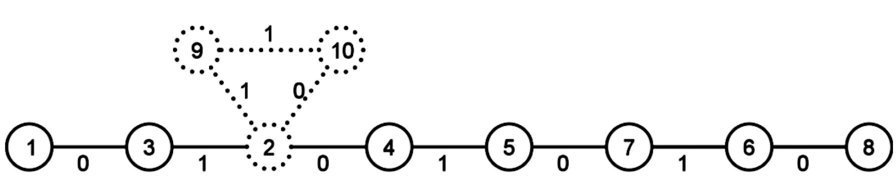

Fig 1(b) Balanced DSD cordial graph G' (by duplicating)

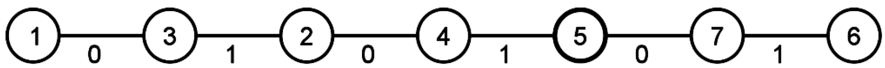

Fig 2(a) Balanced DSD cordial graph $G(P_7)$

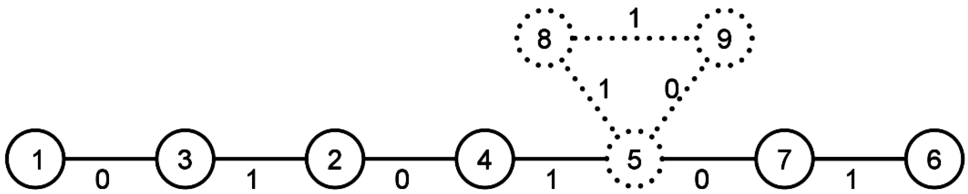

Fig 2(b) Unbalanced DSD cordial graph G' (by duplicating)

Proposition 3.5

A graph got by duplicating a vertex v_k with an edge $e' = u'v'$ in a DSD cordial cycle $C_n \left(n \equiv 0,1(mod\,4)\right)$ is DSD cordial.

Proof

Let G be a cycle graph $C_n \left(n \equiv 0,1(mod\,4)\right)$. By Proposition 2.6, we draw a DSD cordial cycle graph C_n. Now, we duplicate any of the vertex v_k in G with an edge $e' = u'v'$ and construct a new graph G'. In this graph $G', \left|V\left(G'\right)\right| = n+2$ and $\left|E\left(G'\right)\right| = n+3$. For DSD cordial labeling pattern, let the vertex labels are $\{1,2,\ldots,n+2\}$. Then, by labeling u' and v' by $f\left(u'\right) = n+1$ and $f(v') = n+2$, we get $\left|e_f(0) - e_f(1)\right| \le 1$.

Hence G' is also a DSD cordial.

Remark 3.6

From Proposition 3.5, in particular we get $\left|e_f(0) - e_f(1)\right| = 0$ when n is odd.

Hence, we can conclude that G' is a balanced DSD cordial graph when n is odd and unbalanced DSD cordial graph when n is even.

Note 3.7

For $n \equiv 3(mod\,4)$, the cycle graph C_n is DSD cordial whereas G' obtained by duplicating any of the vertex with an edge in C_n is not DSD cordial.

Example 3.8

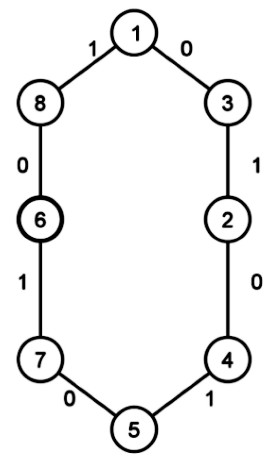

Fig 3(a) Balanced DSD cordial graph $G(C_8)$

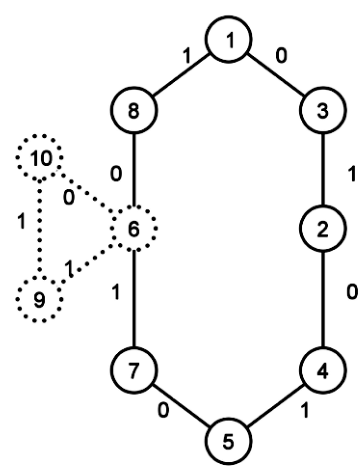

Fig 3(b) Unbalanced DSD cordial graph G' (by duplicating v_7)

Proposition 3.9

A graph got by duplicating a vertex $v_k\left(1 \le k \le n\right)$ with an edge $e' = u'v'$ in a DSD cordial graph $K_{1,n}$ (n is even) is DSD cordial.

Proof

Let G be a star graph $K_{1,n}$ (n is even). By Proposition 2.6, we draw a DSD cordial star graph $K_{1,n}$. Now, we duplicate any of the vertex v_k in G with an edge $e' = u'v'$ and construct a new graph G' . In this graph $G', \left|V\left(G'\right)\right| = n+3$ and $\left|E\left(G'\right)\right| = n+3$. For DSD cordial labeling pattern, let the vertex labels are $\{1,2,\ldots,n+3\}$. Then, by labeling u' and v' by $f\left(u'\right) = n+2$ and $f(v') = n+3$, we get $\left|e_f\left(0\right) - e_f\left(1\right)\right| \le 1$.
Hence G' is also a DSD cordial.

Remark 3.10

From Proposition 3.9, in particular we get $\left|e_f\left(0\right) - e_f\left(1\right)\right| = 1$. Hence, we can conclude that G' is a unbalanced DSD cordial.

Example 3.11

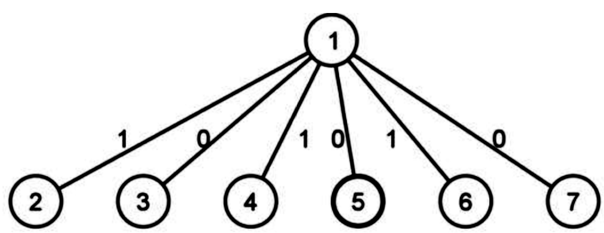

Fig 4(a) Balanced DSD cordial graph $G(K_{1,6})$

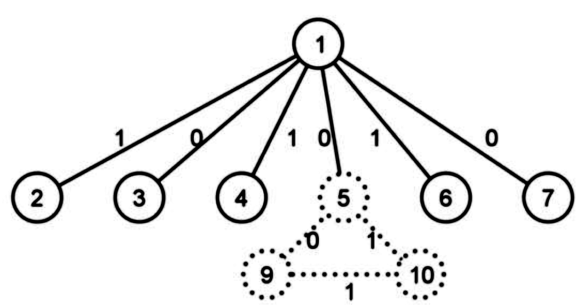

Fig 4(b) Balanced DSD cordial graph G' (by duplicating v_4)

Proposition 3.12

A graph got by duplicating a vertex $v_k\left(1 \le k \le n\right)$ with an edge $e' = u'v'$ in a DSD cordial wheel graph $W_n\left(n \equiv 0,1\left(mod\,4\right)\right)$ is DSD cordial.

Proof

Let G be a DSD wheel graph W_n . By Proposition 2.7, we draw a DSD cordial wheel graph W_n . Now, we duplicate any of the vertex v_k in G with an edge $e' = u'v'$ and construct a new graph G' . In this graph $G', \left|V\left(G'\right)\right| = n+3$ and $\left|E\left(G'\right)\right| = 2n+3$. For DSD cordial labeling pattern, let the vertex labels are $\{1,2,\ldots,n+3\}$. Then, by labeling u' and v' by $f\left(u'\right) = n+2$ and $f(v') = n+3$, we get $\left|e_f\left(0\right) - e_f\left(1\right)\right| \le 1$.

Hence G' is also a DSD cordial.

Remark 3.13

From Proposition 3.12, in particular we get $\left|e_f\left(0\right) - e_f\left(1\right)\right| = 1$. Hence, we can conclude that G' is a unbalanced DSD cordial graph.

Example 3.14

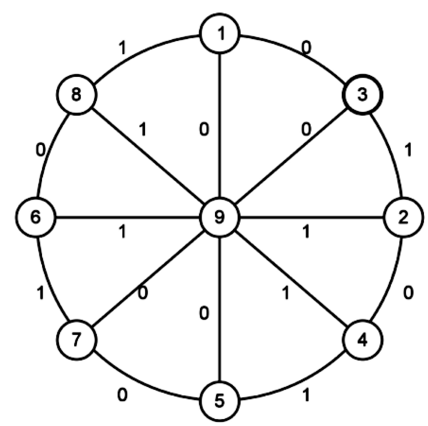

Fig 5(a) Balanced DSD cordial graph $G(W_8)$

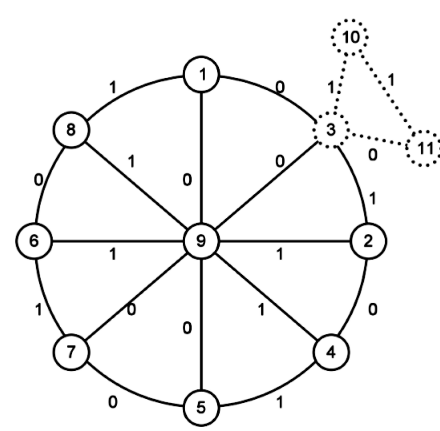

Fig 5(b) Unbalanced DSD cordial graph G' (by duplicating v_2)

Proposition 3.15

A graph got by duplicating a vertex v_k with an edge $e' = u'v'$ in a DSD cordial helm graph $H_n\left(n \equiv 0,1(mod\,4)\right)$ is DSD cordial.

Proof

Let G be a helm graph H_n. By Proposition 2.7, we draw a DSD cordial helm graph H_n. Now, we duplicate any of the vertex v_k (either rim vertices or apex vertex or pendent vertices) in G with an edge $e' = u'v'$ and construct a new graph G'. In this graph $G', \left|V\left(G'\right)\right| = 2n+3$ and $\left|E\left(G'\right)\right| = 3n+3$. For DSD cordial labeling pattern, let the vertex labels are $\{1,2,\ldots,2n+3\}$. Then, by labeling u' and v' by $f\left(u'\right) = 2n+2$ and $f(v') = 2n+3$, we get $\left|e_f(0) - e_f(1)\right| \le 1$.

Hence G' is also a DSD cordial.

Remark 3.16

From Proposition 3.15, in particular we get $\left|e_f(0) - e_f(1)\right| = 0$ when n is odd.

Hence, we can conclude that is a balanced DSD cordial graph when G' and unbalanced DSD cordial graph when n is even.

Example 3.17

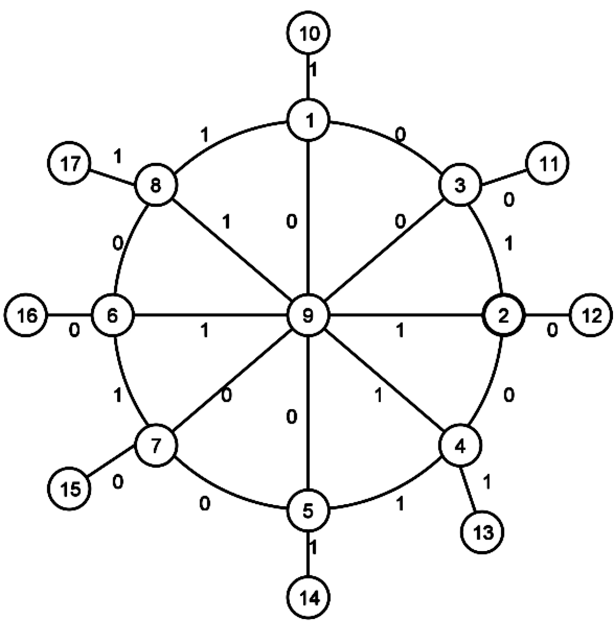

Fig 6(a) Balanced DSD cordial graph $G(H_8)$

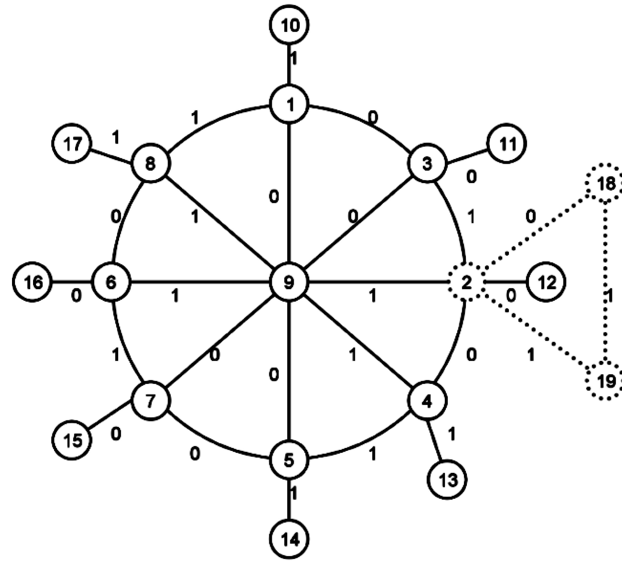

Fig 6(b) Unbalanced DSD cordial graph G' (by duplicating v_3)

Proposition 3.18

A graph got by duplicating a vertex v_k with an edge $e' = u'v'$ in a DSD cordial crown graph $C_n \odot K_1$ is divided square difference cordial.

Proof

Let G be a crown graph $C_n \odot K_1$. By Proposition 2.8, we draw a DSD cordial crown graph $C_n \odot K_1$. Now, we duplicate any of the vertex v_k (either vertices in the cycle or pendent vertices) in G with an edge $e' = u'v'$ and construct a new graph G'. In this graph $G', \left|V\left(G'\right)\right| = 2n+2$ and $\left|E\left(G'\right)\right| = 2n+3$. For DSD cordial labeling pattern, let the vertex labels are $\{1,2,\ldots,2n+2\}$. Then, by labeling u' and v' by $f\left(u'\right) = 2n+1$ and $f(v') = 2n+2$, we get $\left|e_f(0) - e_f(1)\right| \le 1$.

Hence G' is also a DSD cordial.

Remark 3.19

From Proposition 3.18, in particular we get $\left|e_f(0) - e_f(1)\right| = 1$.

Hence, we can conclude that G' is a unbalanced DSD cordial graph.

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Example 3.20

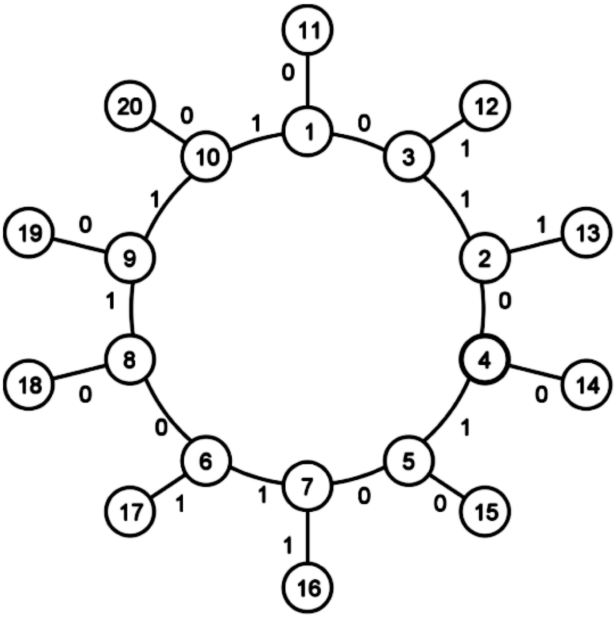

Fig 7(a) Balanced DSD cordial graph G $(C_{10} \odot K_1)$

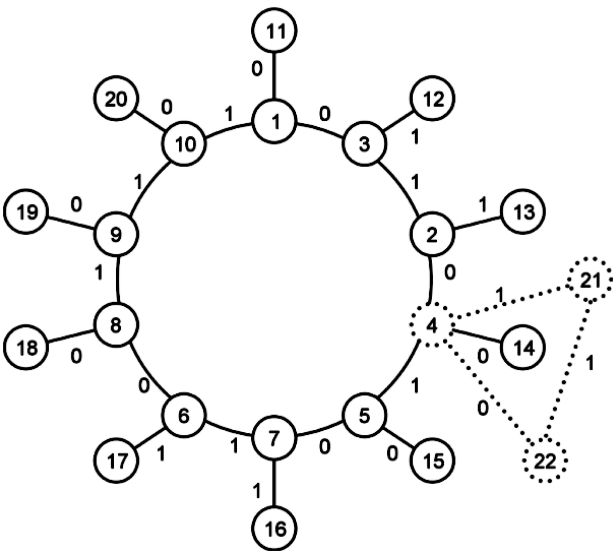

Fig 7(b) Unbalanced DSD cordial graph G' (by duplicating v_4)

Proposition 3.21

A graph got by duplicating a vertex v_k with an edge $e' = u'v'$ in a DSD cordial comb graph $P_n \odot K_1$ (except $n \equiv 1 \, mod\, 4$) is divided square difference cordial.

Proof

Let G be a comb graph $P_n \odot K_1$ (except $\mathrm{n} \equiv 1 mod 4$). By Proposition 2.8, we draw a DSD cordial comb graph $P_n \odot K_1$. Now, we duplicate any of the vertex v_k (either in path or pendent vertex) in G with an edge $e' = u'v'$ and construct a new graph G'. In this graph $G', |V(G')| = 2n+2$ and $|E(G')| = 2n+2$. For DSD cordial labeling pattern, let the vertex labels are $\{1, 2, \dots, 2n+2\}$. Then, by labeling u' and v' by $f(u') = 2n+1$ and $f(v') = 2n+2$, we get $|e_f(0) - e_f(1)| \le 1$.

Hence G' is also a DSD cordial.

Remark 3.22

From Proposition 3.21, in particular we get $|e_f(0) - e_f(1)| = 0$.

Hence, we can conclude that G' is a balanced DSD cordial graph.

Note 3.23

For $n \equiv 1 (mod 4)$, the comb graph $P_n \odot K_1$ is DSD cordial whereas G' obtained by duplicating any of the vertex with an edge in $P_n \odot K_1$ is not DSD cordial.

Example 3.24

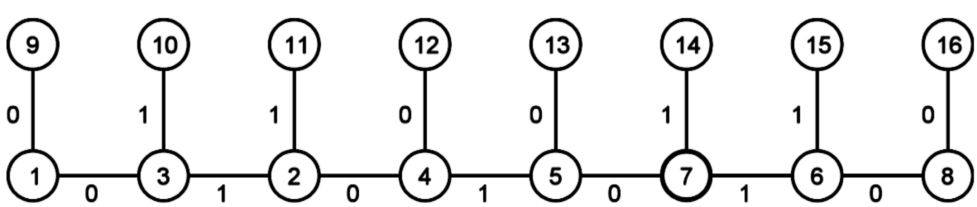

Fig 8(a) Unbalanced DSD cordial graph $G(P_8 \odot K_1)$

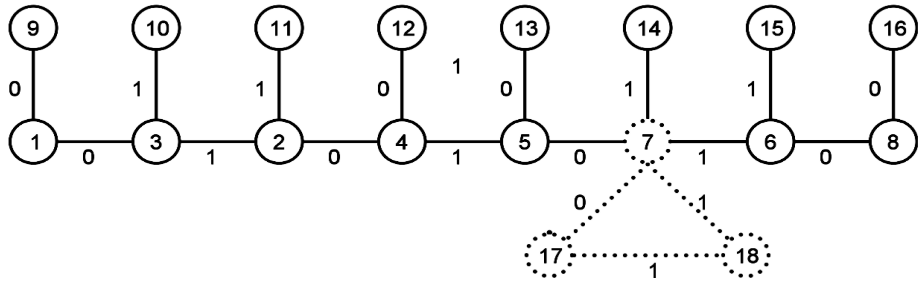

Fig 8(b) Balanced DSD cordial graph G' (by duplicating v_6)

Proposition 3.25

A graph got by duplicating a vertex v_k with an edge $e' = u'v'$ in a DSD cordial triangular snake graph T_n (except $n \equiv 2\bmod 4$) is divided square difference cordial.

Proof

Let G be a triangular snake graph T_n (except $n \equiv 3 \bmod 4$). By Proposition 2.9, we draw a DSD cordial triangular snake graph T_n. Now, we duplicate any of the vertex v_k (either vertex in path or triangle) in G with an edge $e' = u'v'$ and construct a new graph G'. In this graph G', $\left|V\left(G'\right)\right| = 2n+1$ and $\left|E\left(G'\right)\right| = 3n$. For DSD cordial labeling pattern, let the vertex labels are $\{1, 2, \ldots, 2n+1\}$. Then, by labeling u' and v' by $f\left(u'\right) = 2n$ and $f(v') = 2n+1$, we get $\left|e_f(0) - e_f(1)\right| \le 1$.

Hence G' is also a DSD cordial.

Remark 3.26

From Proposition 3.25, in particular we get $\left|e_f(0) - e_f(1)\right| = 0$ when n is even.

Hence we can conclude that G' is a balanced DSD cordial graph.

Note 3.27

1. For $n \equiv 2(mod\, 4)$, the triangular snake graph T_n is DSD cordial whereas G' got by duplicating any of the vertex with an edge in T_n is not DSD cordial.

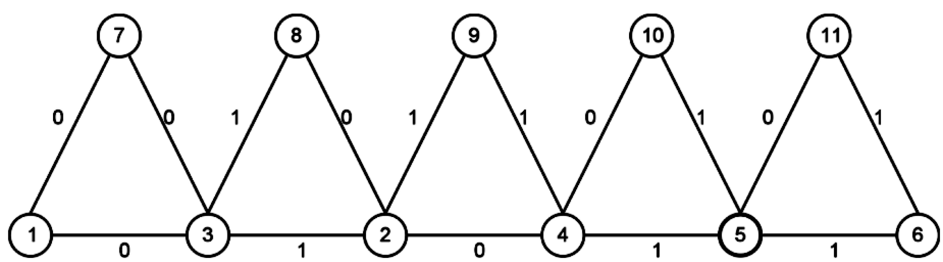

Fig 9(a) Unbalanced DSD cordial graph $G(T_6)$

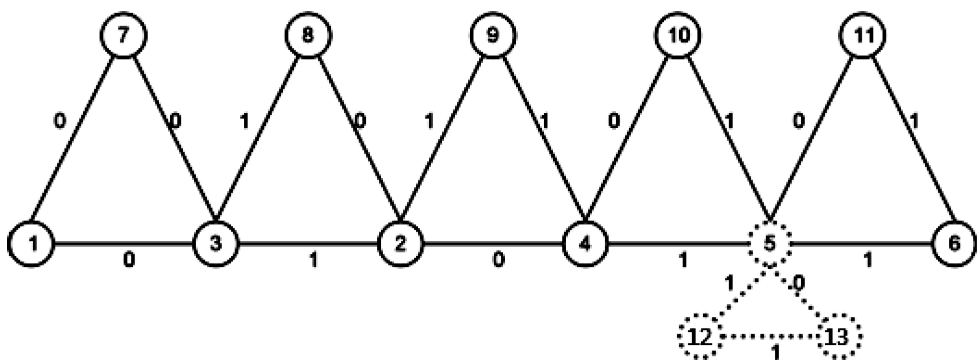

Fig 9(b) Balanced DSD cordial graph G' (by duplicating v_5)

2. For $n \equiv 3(mod\,4)$, the triangular snake graph T_n is not DSD cordial whereas G' got by duplicating any of the vertex with an edge in T_n is DSD cordial.

4. Conclusion

In this article, we have discussed and proven that the graph got by duplicating a vertex with an edge in divided square difference (DSD) cordial graphs such as path graph, cycle graph, star graph, wheel graph, helm graph, bistar graph, crown graph, comb graph and snake graph were also DSD cordial graphs.

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